Electrostatic Potential and Capacitance

1. Ten capacitors, each of capacitance 1 μF, are connected in parallel to a source of 100 V. The total energy stored in the system is equal to: (2024)

(A) 10^{-2} J

(B) 10^{-3} J

(C) 0.5×10^{-3} J

(D) 5.0×10^{-2} J

Ans. (D) 5.0×10^{-2} J

2.2 Electrostatic Potential

VSA (1 mark)

1. The physical quantity having SI unit $NC^{-1}m$ is ______. *(2020)* R

2.3 Potential due to a Point Charge

VSA (1 mark)

2. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero?

(Delhi 2016, Foreign 2016)

2.4 Potential due to an Electric Dipole

SA II (3 marks)

3. Derive the expression for the electric potential due to an electric dipole at a point on its axial line. *(2/3, Delhi 2017)* Ap

2.5 Potential due to a System of Charges

SA I (2 marks)

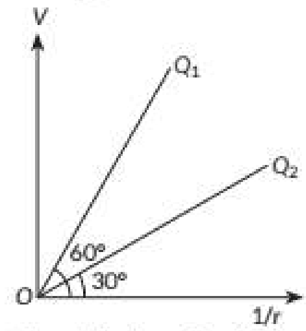
4. Two small conducting balls A and B of radius r_1 and r_2 have charges q_1 and q_2 respectively. They are connected by a wire. Obtain the expression for charges on A and B, in equilibrium. *(2023)*

5. N small conducting liquid droplets, each of radius r, are charged to a potential V each. These droplets coalesce to form a single large drop without any charge leakage. Find the potential of the large drop. *(2020)* An

Answer the following questions based on the above :

(a) Consider a uniformly charged thin conducting shell of radius R. Plot a graph showing the variation of $|\vec{E}|$ with distance r from the centre, for points $0 \le r \le 3R$.

(b) The figure shows the variation of potential V with $\frac{1}{r}$ for two point charges Q_1 and Q_2, where V is the potential at a distance r due to a point charge. Find $\frac{Q_1}{Q_2}$



(c) An electric dipole of dipole moment of 6×10^{-7} C-m is kept is a uniform electric field of 10^4 N/C such that the dipole moment and the electric field are parallel. Calculate the potential energy of the dipole.

OR

An electric dipole of dipole moment $\vec{p}$ is initially kept in a uniform electric field $\vec{E}$ such that $\vec{p}$ is perpendicular to $\vec{E}$. Find the amount of work done in rotating the dipole to a position at which $\vec{p}$ becomes antiparallel to $\vec{E}$. *(2023)*

6. Two point charges q and $-2q$ are kept 'd' distance apart. Find the location of point relative to charge 'q' at which potential due to this system of charges is zero. *(AI 2014C)*

LA (4 marks)

The following questions are source based/case based questions. Read the case carefully and answer the questions that follow.

7. Electrostatics deals with the study of forces, fields and potentials arising from static charges. Force and electric field, due to a point charge is basically determined by Coulomb's law. For symmetric charge configurations, Gauss's law, which is also based on Coulomb's law, helps us to find the electric field. A charge/ a system of charges like a dipole experience a force/torque in an electric field. Work is required to be done to provide a specific orientation to a dipole with respect to an electric field.

OR

"For any charge configuration, equipotential surface through a point is normal to the electric field." Justify. *(Delhi 2014)*

SA II (3 marks)

11. (a) Draw the equipotential surfaces corresponding to a uniform electric field in the z-direction.
(b) Derive an expression for the electric potential at any point along the axial line of an electric dipole. *(AI 2019)*

12. Draw the equipotential surface due to an electric dipole. *(1/3, Delhi 2019)*

OR

Depict the equipotential surfaces due to an electric dipole. *(2/3, Delhi 2017)*

13. Define an equipotential surface. Draw equipotential surfaces:
(i) in the case of a single point charge and
(ii) in a constant electric field in Z-direction.
Why the equipotential surface about a single charge are not equidistant ?
(iii) Can electric field exist tangential to an equipotential surface? Give reason.
(AI 2016) R

14. Two closely spaced equipotential surfaces A and B with potentials V and $V + \delta V$, (where δV is the change in V), are kept δl distance apart as shown in

2.6 Equipotential Surfaces

MCQ

8. The electric potential V at any point (x, y, z) is given by $V = 3x^2$ where x is in metres and V in volts. The electric field at the point (1 m, 0, 2 m) is
(a) 6 V m^{-1} along $-x$-axis
(b) 6 V m^{-1} along $+x$-axis
(c) 1.5 V m^{-1} along $-x$-axis
(d) 1.5 V m^{-1} along $+x$-axis. *(Term I 2021-22)* U

9. Equipotentials at a large distance from a collection of charges whose total sum is not zero are
(a) spheres (b) planes
(c) ellipsoids (d) paraboloids
(Term I 2021-22) R

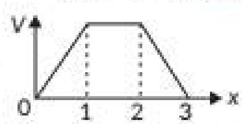
VSA (1 mark)

10. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor?
(AI 2015C) U

17. Write two important characteristics of equipotential surfaces. *(2/5, 2020)*

18. The magnitude of electric field (in N C^{-1}) in a region varies with the distance r (in m) as
$E = 10r + 5$
By how much does the electric potential increase in moving from point at r = 1 m to a point at r = 10 m.
(2/5, 2020) Ap

19. The electric potential as a function of distance 'x' is shown in the figure. Draw a graph of the electric field E as a function of x. *(1/5, AI 2019)* An



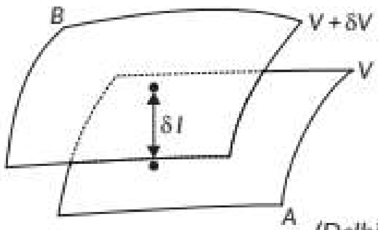
20. Is the electrostatic potential necessarily zero at a point where the electric field is zero? Give an example to support your answer.
(2/5, AI 2019) Ap

2.7 Potential Energy of a System of Charges

MCQ

21. A + 3.0 nC charge Q is initially at rest at a distance of r_1 = 10 cm from a + 5.0 nC charge q fixed at the origin. The charge Q is moved away from q to a new position at r_2 = 15 cm. In this process work done by the field is
(a) 1.29×10^{-5} J (b) 3.6×10^{5} J
(c) -4.5×10^{-7} J (d) 4.5×10^{-7} J

the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potentials.

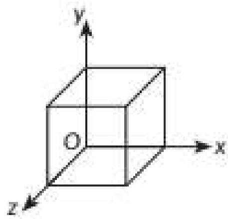


(Delhi 2014C) Ev

LA (5 marks)

15. Draw equipotential surfaces due to an isolated point charge ($-q$) and depict the electric field lines. (AI 2020)

16. A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m.



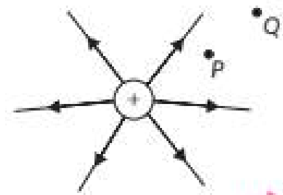
Find the
(i) electric field $\vec{E}$ and
(ii) total electric flux through the cube.

(3/5, 2020) An

(Term I 2021-22)

VSA (1 mark)

22. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason.

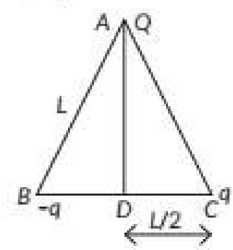


(Foreign 2014) U

SA I (2 marks)

23. Obtain an expression for electrostatic potential energy of a system of three charges q, $2q$ and $-3q$ placed at the vertices of an equilateral triangle of side a. (2023)

24. Three point charges Q, q and $-q$ are kept at the vertices of an equilateral triangle of side L as shown in figure. What is
(i) the electrostatic potential energy of the arrangement? and
(ii) the potential at point D?



(2023)

25. Two point charges q and $-q$ are located at $(0, 0, -a)$ and $(0, 0, a)$ respectively.
(a) Depict the equipotential surfaces due to this arrangement.
(b) Find the amount of work done in moving a small test charge q_0 from point $(l, 0, 0)$ to $(0, 0, 0)$.

(AI 2020C)

26. Obtain the expression for potential energy of an electric dipole placed with its axis at an angle (θ) to an external electric field ($\vec{E}$). What is the minimum value of the potential energy? (AI 2019C)

SA II (3 marks)

27. (a) Two point charges $+Q_1$ and $-Q_2$ are placed r distance apart. Obtain the expression for the amount of work done to place a third charge Q_3 at the midpoint of the line joining the two charges.
(b) At what distance from charge $+Q_1$ on the line joining the two charges (in terms of Q_1, Q_2 and r)

LA (5 marks)

33. Find the expression for the potential energy of a system of two point charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$, respectively in an external electric field $\vec{E}$.

(2/5, 2020) Ap

34. Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges. (2/5, 2020) An

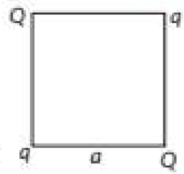
35. Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

(3/5, AI 2019)

36. An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet. (3/5, AI 2017) Ap

will this work done be zero. (2020) Ev

28. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure. Find the



(a) resultant electric force on a charge Q, and
(b) potential energy of this system. (2018) An

29. (a) Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.
(b) Find out the amount of the work done to separate the charges at infinite distance. (2018)

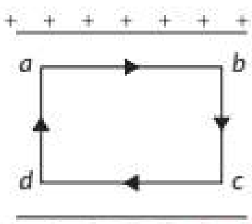
30. Three point charges +1 μC, −1 μC and +2 μC are initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10 cm. (2017)

2.8 Potential Energy in an External Field

SA I (2 marks)

31. Obtain the expression for potential energy of an electric dipole placed with its axis at an angle (θ) to an external electric field ($\vec{E}$). What is the minimum value of the potential energy? (2019C)

32. The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop $abcda$.



(Delhi 2014) U

LA (5 marks)

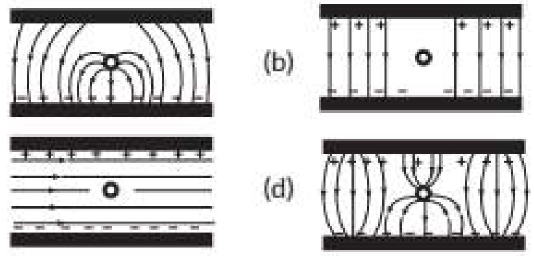
40. When a parallel plate capacitor is connected across a dc battery, explain briefly how the capacitor gets charged. (2/5, AI 2019) U

41. If two similar large plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, find the expressions for
(a) field at points between the two plates and on outer side of the plates. Specify the direction of the field in each case.
(b) the potential difference between the plates.
(c) the capacitance of the capacitor so formed. (3/5, AI 2016)

2.9 Electrostatics of Conductors

MCQ

37. Which of the diagrams correctly represents the electric field between two charged plates if a neutral conductor is placed in between the plates?



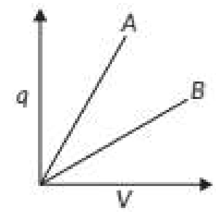
(a) (b)
(c) (d)

(Term I 2021-22)

2.11 Capacitors and Capacitance

VSA (1 mark)

38. The given graph shows variation of charge 'q' versus potential difference 'V' for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that of C_1. Which line (A or B) corresponds to C_1 and why? (AI 2014C) U



2.12 The Parallel Plate Capacitor

MCQ

39. A charge particle is placed between the plates of a charged parallel plate capacitor. It experiences a force F. If one of the plates is removed, the force on the charge particle becomes
(a) F (b) 2F
(c) F/2 (d) Zero (AI 2020C)

(i) Calculate the capacitance of the capacitor.
(ii) If this capacitor is connected to 100 V supply, what would be the charge on each plate?
(iii) How would charge on the plates be affected, if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected? (Foreign 2014) Ev

LA (4 marks)

46. A capacitor is a system of two conductors separated by an insulator. The two conductors have equal and opposite charges with a potential difference between them. The capacitance of a capacitor

2.13 Effect of Dielectric on Capacitance

MCQ

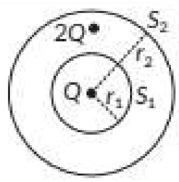
42. Two capacitors of capacitances C_1 and C_2 are connected in parallel. If a change Q is given to the combination, the ratio of the charge on the capacitor C_1 to the charge on C_2 will be:

(a) C_1/C_2 (b) $\sqrt{\frac{C_1}{C_2}}$ (c) $\sqrt{\frac{C_2}{C_1}}$ (d) $\frac{C_2}{C_1}$

(AI 2020)

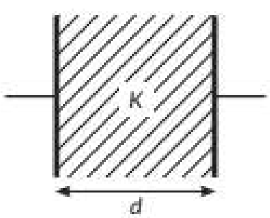
SA I (2 marks)

43. A sphere S_1 of radius r_1 encloses a net charge Q. If there is another concentric sphere S_2 of radius $r_2 (r_2 > r_1)$ enclosing charge $2Q$, find the ratio of the electric flux through S_1 and S_2. How will the electric flux through sphere S_1 change if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air? *(AI 2014C)* Ap

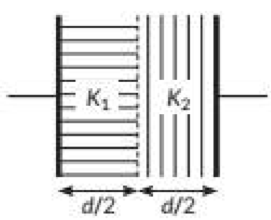


SA II (3 marks)

44. The space between the plates of a parallel plate capacitor is completely filled in two ways. In the first case, it is filled with a slab of dielectric constant K. In the second case, it is filled with two slabs of equal thickness and dielectric constants K_1 and K_2 respectively as shown in the figure. The capacitance of the capacitor is same in the two cases. Obtain the relationship between K, K_1 and K_2.



(Case 1)



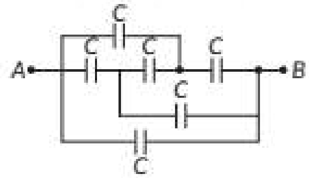
(Case 2)

(AI 2020)

45. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the separation between the plates is 0.3 mm.

depends on the geometrical configuration (shape, size and separation) of the system and also on the nature of the insulator separating the two conductors. They are used to store charges. Like resistors, capacitors can be arranged in series or parallel or a combination of both to obtain desired value of capacitance.

(i) Find the equivalent capacitance between points A and B in the given diagram.

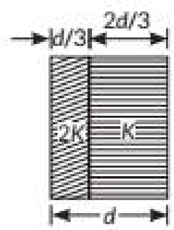


(ii) A dielectric slab is inserted between the plates of a parallel plate capacitor. The electric field between the plates decreases. Explain.

(iii) A capacitor A of capacitance C, having charge Q is connected across another uncharged capacitor B of capacitance $2C$. Find an expression for (a) the potential difference across the combination and (b) the charge lost by capacitor A.

OR

(iii) Two slabs of dielectric constants $2K$ and K fill the space between the plates of a parallel plate capacitor of plate area A and plate separation d as shown in figure. Find an expression for capacitance of the system.



(2023)

2.14 Combination of Capacitors

MCQ

47. Three capacitors, each of 4 μF are to be connected in such a way that the effective capacitance of the combination is 6 μF. This can be achieved by connecting.

(a) All three in parallel
(b) All three in series
(c) Two of them connected in series and the combination in parallel to the third.
(d) Two of them connected in parallel and the combination in series to the third. *(2023)*

2.15 Energy Stored in a Capacitor

MCQ

48. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from 2 μF to X μF, the decrease in energy of the capacitor is

54. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination? If these were connected in parallel across the same battery, how much energy will be stored in the combination now?

2×10^{-2} J. The value of X is

(a) 1 μF (b) 2 μF (c) 3 μF (d) 4 μF

(Term I 2021-22)

SA I (2 marks)

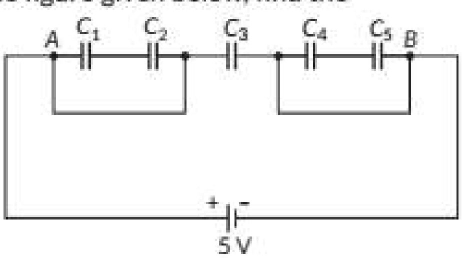
49. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V, C_1 = 20 μF, C_2 = 30 μF and C_3 = 15 μF.

(AI 2015) An

50. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. *(AI 2014)*

SA II (3 marks)

51. In the figure given below, find the

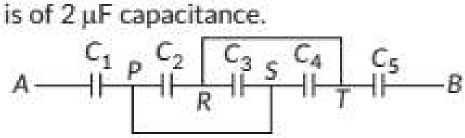


(a) equivalent capacitance of the network between points A and B.
Given : $C_1 = C_5$ = 4 μF, $C_2 = C_3 = C_4$ = 2 μF.

(b) maximum charge supplied by the battery, and

(c) total energy stored in the network. *(2020)* An

52. (i) Find the equivalent capacitance between A and B in the combination given below. Each capacitor is of 2 μF capacitance.



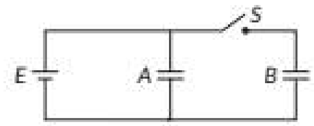
(ii) If a dc source of 7 V is connected across AB, how much charge is drawn from the source and what is the energy stored in the network?

(Delhi 2017)

53. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. *(Delhi 2017)* Ap

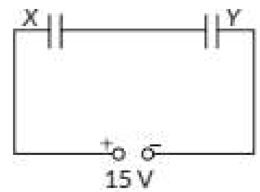
Also find the charge drawn from the battery in each case. *(Delhi 2017)*

55. Two identical parallel plate capacitors A and B are connected to a battery of V volt with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



(AI 2017) An

56. Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric of $\varepsilon_r = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Calculate the potential difference between the plates of X and Y.

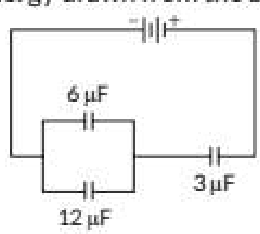
(iii) Estimate the ratio of electrostatic energy stored in X and Y. *(Delhi 2016)*

57. In the following arrangement of capacitors, the energy stored in the 6 μF capacitor is E. Find the value of the following

(i) Energy stored in 12 μF capacitor

(ii) Energy stored in 3 μF capacitor

(iii) Total energy drawn from the battery



(Foreign 2016) Ap

58. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination. *(Delhi 2015)* Cr

LA **(5 marks)**

59. (i) (A) Why does the electric field inside a dielectric slab decrease when kept in an external electric field ?

(B) Derive an expression for the capacitance of a parallel plate capacitor filled with a medium of dielectric constant K.

(ii) A charge $q = 2\ \mu C$ is placed at the centre of a sphere of radius 20 cm.
What is the amount of work done in moving $4\ \mu C$ from one point to another point on its surface ?

(iii) Write a relation for polarisation $\vec{P}$ of a dielectric material in the presence of an external electric field. *(AI 2021C)*

60. (i) Obtain an expression for the potential energy of an electric dipole placed in a uniform electric field.

(ii) Three capacitors of capacitance C_1, C_2 and C_3 are connected in series to a source of V volt. Show that the total energy stored in the combination of capacitors is equal to sum of the energy stored in individual capacitors.

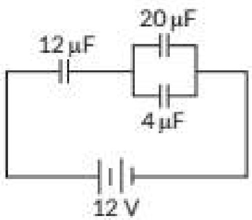
(iii) A capacitor of capacitance C is connected across a battery. After charging, the battery is disconnected and the separation between the plates is doubled. How will

(i) the capacitance of the capacitor, and

(ii) the electric field between the plates be affected? Justify your answer. *(AI 2021C)*

61. (a) Obtain the expressions for the resultant capacitance when the three capacitors C_1, C_2 and C_3 are connected (i) in parallel and then (ii) in series.

(b) In the circuit shown in the figure, the charge on the capacitor of $4\ \mu F$ is $16\ \mu C$. Calculate the energy stored in the capacitor of $12\ \mu F$ capacitance.



(AI 2019C)

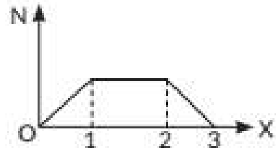
62. (a) When a parallel plate capacitor is connected across a dc battery, explain briefly how the capacitor gets changed.

(b) A parallel plate capacitor of capacitance 'C' is changed to V volt by a battery. After some time the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant $1 < K < 2$ is introduced to full the space between the plates. How will the following be affected?

(i) The electric field between the plates of the capacitor.

(ii) The energy stored in the capacitor.
Justify your answer in each case

(iii) The electric potential as a function of distance x is shown in the following figure. Draw a graph of the electric field F as a function of x.



(AI 2019)

63. A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor. *(2/5, Delhi 2019)*

64. A parallel plate capacitor of capacitance 'C' is charged to 'V' volt by a battery. After some time the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant $1 < K < 2$ is introduced to fill the space between the plates. How will the following be affected?

(i) The electric field between the plates of the capacitor?

(ii) The energy stored in the capacitor.

Justify your answer in each case. *(2/5, AI 2019)* Ev

65. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same. *(3/5, AI 2016)* Ev

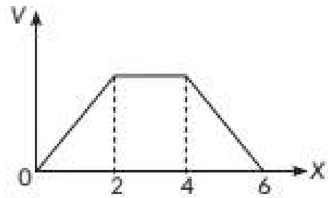
66. A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor. *(AI 2015)*

CBSE Sample Questions

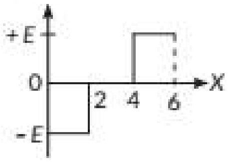
2.2 Electrostatic Potential

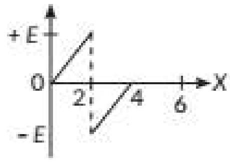
MCQ

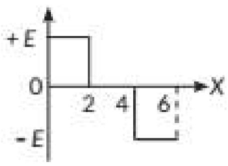
1. The electric potential V as a function of distance X is shown in the figure.

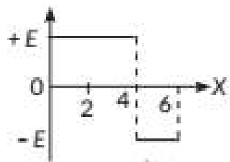


The graph of the magnitude of electric field intensity E as a function of X is

(a) 

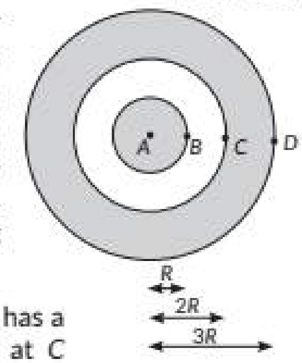
(b) 

(c) 

(d) 

(2022-23)

2. A solid spherical conductor has charge $+Q$ and radius R. it is surrounded by a solid spherical shell with charge $-Q$, inner radius $2R$, and outer radius $3R$. Which of the following statements is true?



(a) The electric potential has a maximum magnitude at C and the electric field has a maximum magnitude at A.
(b) The electric potential has a maximum magnitude at D and the electric field has a maximum magnitude at B.
(c) The electric potential at A is zero and the electric field has a maximum magnitude at D.
(d) Both the electric potential and electric field achieve a maximum magnitude at B.

(Term I 2021-22) Ap

2.5 Potential due to a System of Charges

MCQ

3. The electric potential on the axis of an electric dipole at a distance 'r' from it's centre is V. Then the potential at a point at the same distance on its equatorial line will be

(a) $2V$ (b) $-V$ (c) $\frac{V}{2}$ (d) Zero

(a) They do not cross each other.
(b) The rate of change of potential with distance on them is zero.
(c) For a uniform electric field they are concentric spheres.
(d) They can be imaginary spheres.

(Term I 2021-22) U

MCQ

For question number 6, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

6. **Assertion (A) :** Electric field is always normal to equipotential surfaces and along the direction of decreasing order of potential
Reason (R) : Negative gradient of electric potential is electric field.
(a) Both A and R are true and R is correct explanation of A.
(b) Both A and R are true, and R is not correct explanation of A.
(c) A is true, but R is false.
(d) A is false and R is also false. (2020-21)

SA I **(2 marks)**

7. Establish the relation between electric field and electric potential at a point. Draw the equipotential surface for an electric field pointing in $+Z$ direction with its magnitude increasing at constant rate along $-Z$ direction. (2020-21) U

2.7 Potential Energy of a System of Charges

MCQ

8. An electric dipole of moment p is placed parallel to the uniform electric field. The amount of work done in rotating the dipole by 90° is
(a) $2pE$ (b) pE (c) $pE/2$ (d) zero

(Term I 2021-22)

Given below are two statements labelled as Assertion

(2022-23)

4. Three charges $2q$, $-q$ and $-q$ lie at vertices of a triangle. The value of E and V at centroid of triangle will be
   (a) $E \neq 0$ and $V \neq 0$ (b) $E = 0$ and $V = 0$
   (c) $E \neq 0$ and $V = 0$ (d) $E = 0$ and $V \neq 0$
   (Term I 2021-22)

2.6 Equipotential Surfaces

MCQ

5. Which of the following is NOT the property of equipotential surface?

(A) and Reason (R).

9. **Assertion (A) :** An electron has a high potential energy when it is at a location associated with a more negative value of potential, and a low potential energy when at a location associated with a more positive potential.
   Reason (R) : Electrons move from a region of higher potential to region of lower potential.
   Select the most appropriate answer from the options given below:
   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true but R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) A is false and R is also false. (Term I 2021-22)

SA I (2 marks)

10. Deduce an expression for the potential energy of a system of two point charges q_1 and q_2 located at positions r_1 and r_2 respectively in an external field $(\vec{E})$. (2020-21)

LA (5 marks)

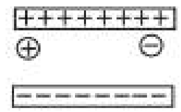
11. (a) Three charges $-q$, Q and $-q$ are placed at equal distances on a straight line. If the potential energy of the system of these charges is zero, then what is the ratio $Q : q$?
    (b) (i) Obtain the expression for the electric field intensity due to a uniformly charged spherical shell of radius R at a point distant r from the centre of the shell outside it.
    (ii) Draw a graph showing the variation of electric field intensity E with r, for $r > R$ and $r < R$.
    (2022-23)

12. (a) Define an ideal electric dipole. Give an example.
    (b) Derive an expression for the torque experienced by an electric dipole in a uniform electric field. What is net force acting on this dipole?
    (c) An electric dipole of length 2 cm is placed with its axis making an angle of 60° with respect to uniform electric field of 10^5 N/C. If it experiences a torque of $8\sqrt{3}$ N/m, calculate the magnitude of charge on the dipole, and its potential energy.
    (2020-21) Ev

2.12 The Parallel Plate Capacitor

MCQ

14. A free electron and a free proton are placed between two oppositely charged parallel plates. Both are closer to the positive plate than the negative plate.

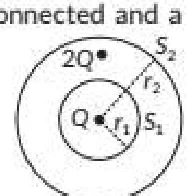


Which of the following statements is true?
I. The force on the proton is greater than the force on the electron.
II. The potential energy of the proton is greater than that of the electron.
III. The potential energy of the proton and the electron is the same.
(a) I only (b) II only
(c) III and I only (d) II and I only
(Term I 2021-22) An

2.13 Effect of Dielectric on Capacitance

MCQ

15. A capacitor plates are charged by a battery with 'V' volts. After charging battery is disconnected and a dielectric slab with dielectric constant 'K' is inserted between its plates, the potential across the plates of the capacitor will become



(a) zero (b) $V/2$
(c) V/K (d) KV (Term I 2021-22) An

LA (5 marks)

16. (a) Draw equipotential surfaces for (i) an electric dipole and (ii) two identical positive charges placed near each other.
    (b) In a parallel plate capacitor with air between the

2.11 Capacitors and Capacitance

MCQ

13. Three capacitors 2 μF, 3 μF and 6 μF are joined in series with each other. The equivalent capacitance is

(a) 1/2 μF (b) 1 μF

(c) 2 μF (d) 11 μF

(Term I 2021-22)

plates, each plate has an area of 6×10^{-3} m^2 and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

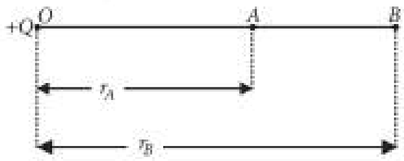
(ii) If the capacitor is connected to 100 V supply, what would be the on each plate?

(iii) How would charge on the plate be affected if a 3 mm thick mica sheet of $k = 6$ is inserted between the plates while the voltage supply remains connected? (2022-23)

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. The physical quantity having SI unit N C^{-1} m is electrostatic potential.

2. 

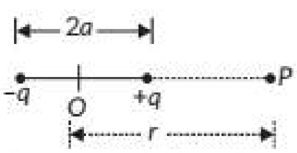
$$r_B > r_A \Rightarrow \frac{1}{r_B} < \frac{1}{r_A} \Rightarrow \left(\frac{1}{r_A} - \frac{1}{r_B}\right) > 0$$

Hence, $(V_A - V_B) > 0$

i.e., potential difference $(V_A - V_B)$ is positive.

Answer Tips

- At infinity, the electric field and the potential are assumed to be zero. So the potential decreases with distance.

3. 

Let P be an axial point at distance r from the centre of the dipole. Electric potential at point P will be

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{r^2-a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \qquad [\because p = q(2a)]$$

For a far away point, $r >> a$

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} \text{ or } V \propto \frac{1}{r^2}$$

Thus, due to a dipole potential at a point is $V \propto \frac{1}{r^2}$.

4. There are two conducting ball having radius r_1 and r_2 and the charge on balls is q_1 and q_2 respectively

Potential difference due to a point charge Q at a distance r is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$\therefore$ From the given figure

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A};\ V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B}$$

$$\therefore\ V_A - V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

5. Let q be the charge on each droplet.

Then $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$ ...(i)

Volume of big drop = N × volume of small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

where R is the radius of the big drop.

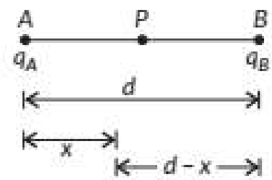
$\Rightarrow\ R = N^{1/3} r$ ...(ii)

and $Q = Nq$, where Q is the charge of bigger drop

$\therefore$ Potential of larger drop,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Nq}{N^{1/3}r} = \frac{N}{N^{1/3}}V = N^{2/3}V$$

6. $q_A = q$ and $q_B = -2q$



$$V_{PA} = \frac{kq_A}{x}$$

$$V_{PB} = \frac{kq_B}{(d-x)}$$

$V_{PA} + V_{PB} = 0$

$\dfrac{kq}{x} = \dfrac{2kq}{(d-x)}$; $d - x = 2x$

$3x = d$; $x = \dfrac{d}{3}$

Concept Applied

- Since there are two charges in the system, the total potential will be given by the superposition equation.

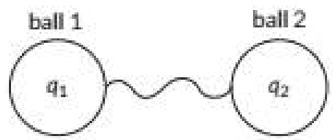
7. (a) Here, a uniformly charged conducting shell of radius R.

So, charge density on ball 1,

$$\sigma_1 = \frac{q_1}{4\pi r_1^2}$$

Charge density on ball 2, $\sigma_2 = \dfrac{q_2}{4\pi r_2^2}$

After connecting for long time, let the charge on ball 1 will be q_1' and charge on ball 2 will be q_2'



After connecting the wire, total charge and area becomes $Q = q_1 + q_2$, area (a) $= 4\pi(r_1^2 + r_2^2)$

Charge density, $\sigma = \dfrac{(q_1+q_2)}{4\pi(r_1^2+r_2^2)}$

So, charge on ball 1, $q_1' = \sigma.\, 4\pi r_1^2$

$$= \frac{(q_1+q_2)}{4\pi}\cdot\frac{4\pi r_1^2}{(r_1^2+r_2^2)}\ ;\ q_1' = \frac{(q_1+q_2)r_1^2}{(r_1^2+r_2^2)}$$

Charge on ball 2, $q_2' = \sigma 4\pi r_2^2$

$$= \frac{(q_1+q_2).4\pi r_2^2}{4\pi(r_1^2+r_2^2)}\ ;\ q_2' = \frac{(q_1+q_2)r_2^2}{(r_1^2+r_2^2)}$$

We know that,

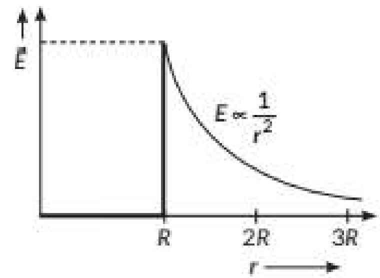
$E_1 = 0, \quad (0 < r < R)$

$E_2 = \dfrac{kQ}{R^2} \quad (r = R)$

$E_3 = \dfrac{kQ}{r^2} \quad (r > R)$

$\left(\because \quad E_2 = \dfrac{kQ}{R^2}\right)$

and at $3R$, $E = \dfrac{kQ}{(3R)^2} = \dfrac{kQ}{9R^2}$



(b) Q_1 has more slope than Q_2.

So, $Q_1 > Q_2$ ...(i)

$V = \dfrac{kQ}{r}$ ...(ii)

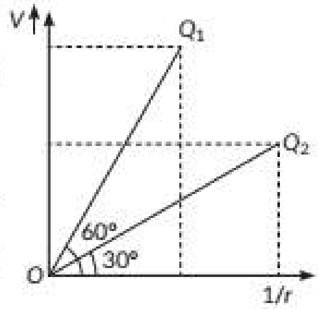
$\left\{V \propto \dfrac{Q}{r}\right\}$

Because both are straight line passing through origin.

$y = mx$ ...(iii)

So, $Q \propto m$,

or $\dfrac{Q_1}{Q_2} = \dfrac{\tan 60°}{\tan 30°} = \dfrac{\sqrt{3}}{1/\sqrt{3}} \quad \therefore \dfrac{Q_1}{Q_2} = \dfrac{3}{1}$



(c) Given, $p = 6 \times 10^{-7}$ C-m

$E = 10^4$ N/C

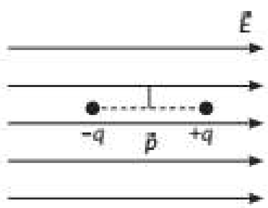
Since, dipole moment and electric field are parallel to each other.

So, $\theta = 0°$.

Potential energy, $U = -pE\cos\theta$

$U = -6 \times 10^{-7} \times 10^4$

$U = -6 \times 10^{-3}$ joules



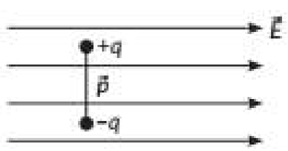
OR

An electric dipole of dipole moment $\vec{p}$ is initially kept in uniform electric field $\vec{E}$,

$U = -PE\cos\theta$

or $\quad dU = -PE\cos\theta\, d\theta$

$$\int dU = -PE\int_{\frac{\pi}{2}}^{-\pi}\cos\theta\, d\theta$$



$$\Delta U = \int dU = -pE\left|\sin\theta\right|_{\frac{\pi}{2}}^{-\pi} = -pE\left[\sin(-\pi) - \sin\frac{\pi}{2}\right]$$

$\therefore \quad \Delta W = \Delta U$

$-\,pE[0 - 1] = pE$

8. (a): Potential, $V = 3x^2$

$$E = \frac{-dV}{dx} = -3\times 2x = -6x$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{r^2-a^2}$$

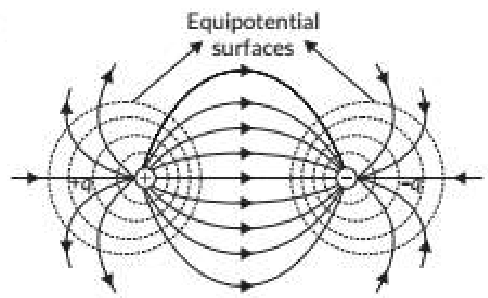
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \qquad [\because p = q\,(2a)]$$

For a far away point, $r >> a$

$\therefore \quad V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{r^2}$ or $V \propto \dfrac{1}{r^2}$

Thus, due to a dipole potential at a point is $V \propto \dfrac{1}{r^2}$.

12.



13. Equipotential surface is the surface with a constant value of potential at all points on the surface.

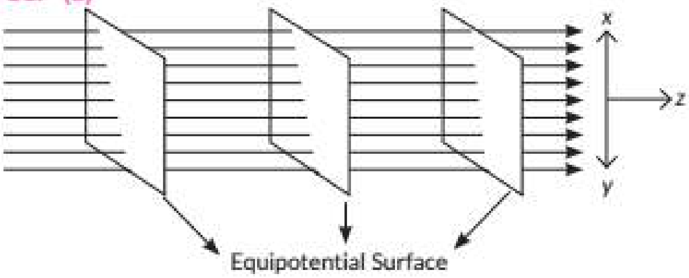
(i) Equipotential surface for single point charge :

$E(1, 0, 2) = -6 \times 1 = -6\ \text{V m}^{-1}$
$E = 6\ \text{V m}^{-1}$ along $-x$ axis

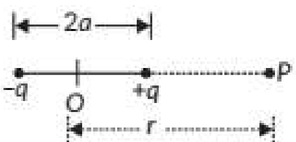
9. (a): For spheres, the equipotentials at a large distance from a collection of charges, the sum is non zero.

10. If the field were not normal to the equipotential surface, it would have a non zero component along the surface. So to move a test charge against this component, a work would have to be done. But there is no potential difference between any two points on an equipotential surface and consequently no work is required to move a test charge on the surface. Hence, the electric field must be normal to the equipotential surface at every point.

11. (a)



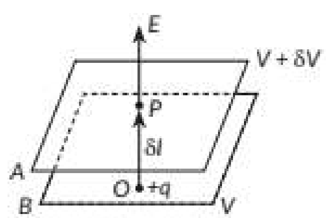
(b)



Let P be an axial point at distance r from the centre of the dipole. Electric potential at point P will be

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$\delta V = \frac{\delta W}{q_0} \qquad \text{...(i)}$$



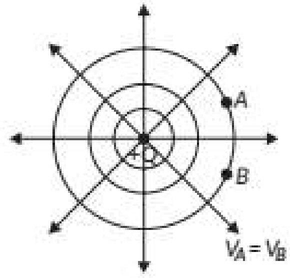
If $\vec{E}$ is electric field at point P due to charge +q placed at point O, then the test charge q_0 experiences a force equal to $q_0\vec{E}$ and the external force required to move the test charge against the electric field $\vec{E}$ is given by

$\vec{F} = -q_0\vec{E}$

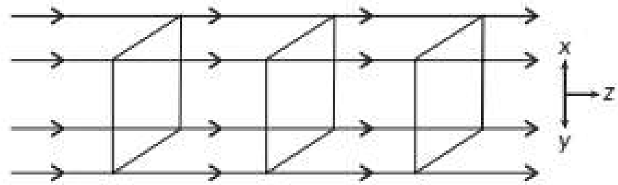
Therefore, work done to move the test charge through an infinitesimally small displacement $\overline{PQ} = \overline{\delta l}$ is given by

$$\Delta W = \vec{F}\cdot\overline{\delta l} = (-q\vec{E})\cdot\overline{\delta l} = -q_0 E\delta l\cos 180^\circ = q_0 E\delta l$$

As the distance r decreases in the direction of δl, then
$\delta W = -q_0 E\,\delta r$



(ii) Equipotential surfaces in a constant electric field in Z-direction.



For constant electric field

Equipotential surfaces about a single charge are not equidistant because electric potential, $V \propto \frac{1}{r}$.

(iii) Electric field cannot exist tangential to an equipotential surface.
If the field lines are tangential, work will be done in moving a charge on the surface whereas on equipotential surface but we know that $W_{AB} = q_0(V_B - V_A) = 0$

14. Electric field as gradient of potential, consider a point charge +q placed at point O. Suppose that V and V+δV are electrostatic potential at surfaces A and B, where distance from the charge +q are r and r − δr respectively.

$$(V + \delta V) = V + \frac{\delta W}{q_0}$$

Answer Tips

- In an uniform electric field, every plane normal to the field direction is an equipotential surface.

17. (a) Properties of equipotential surface are:
(i) Work done in moving a test charge over an equipotential surface is zero.
(ii) Electric field is always directed normal to equipotential surface.

18. Given $E = 10r + 5$

Now the electric potential, $V = -\int E.dr$

$$= -\int_1^{10}(10r+5)dr = -\left[\frac{10r^2}{2} + 5r\right]_1^{10}$$

$$= -1\left[5r^2 + 5r\right]_1^{10} = -[(5\times 100 + 50) - (5+5)] = -540\text{V}$$

19. Electric field $E = -\frac{dV}{dx}$

For $x = 0$ to 1, $V = kx$ ...(i)
$x = 1$ to 2, $V = k$

$$\frac{\delta W}{q_0} = -E\,\delta r \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$\delta V = -E\,\delta r;\ E = -\frac{\delta V}{\delta r}$

Therefore, electric field at a point is equal to the negative gradient of the electrostatic potential at that point.

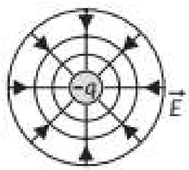
Important conclusions :

(i) No work is done in moving a test charge over an equipotential surface.

(ii) The electric field is always at right angles to the equipotential surface.

(iii) The equipotential surfaces tell the direction of the electric field.

15. For an isolated charge the equipotential surfaces are concentric spherical shells and the separation between consecutive equipotential surfaces increases in the weaker electric field.

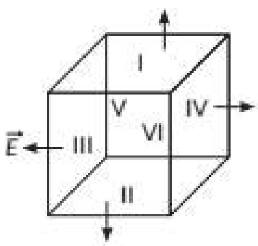


16. (i) Now electric field

$$\vec{E} = -\frac{\partial V}{\partial r} = \frac{-dV}{dx} = \frac{-d}{dx}(10x+5) = -10\hat{i}$$

(ii) Now the total electric flux through the cube,

$\phi = \int E.dS$



$$\phi = \int_{I} E.dS + \int_{II} E.dS + \int_{III} E.dS + \int_{IV} E.dS + \int_{V} E.dS + \int_{VI} E.dS$$

$= 0 + 0 + (+10)(20 \times 10^{-2})^2 + (-10)(20 \times 10^{-2})^2 + 0 + 0 = 0$

Given an equilateral triangle,

Let side of triangle is r.

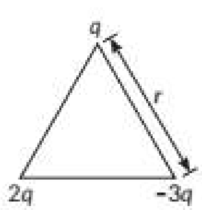
There are three pair of charges. Potential energy,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q(2q)}{r} + \frac{2q(-3q)}{r} + \frac{(-3q)(q)}{r}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{r} - \frac{6q^2}{r} - \frac{3q^2}{r}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{-7q^2}{r}\right]$$

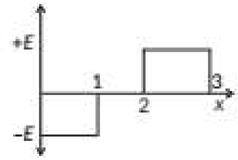
$$U = -\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{r}$$



$x = 2$ to 3, $V = -kx$

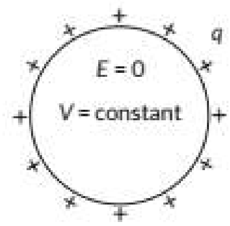
where k is some constant

So, using (i) the variation of electric field is shown in figure.



20. The electric field $E = \frac{-dV}{dr}$

So, even for a constant electric potential electric field can be zero. For example, a hollow shell, the field inside is zero, whereas potential is non-zero and constant.



21. (d): Given, $r_1 = 10$ cm, $r_2 = 15$ cm

Work done = change in PE

$$W = \frac{kqQ}{r_1} - \frac{kqQ}{r_2}$$

$$W = 9\times10^9 \times 5\times3\times10^{-18}\left[\frac{100}{10} - \frac{100}{15}\right] = 9\times15\times10^{-7}\left[\frac{3-2}{30}\right]$$

$$W = \frac{9\times15\times10^{-7}}{30} = 4.5\times10^{-7}\text{ J}$$

22. Work done = q (Potential at Q – Potential at P),

where q is the small positive charge.

The electric potential at a point distance r due to the field created by a positive charge Q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

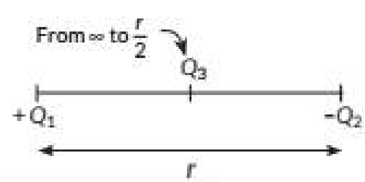
$\because\ r_P < r_Q \ \therefore\ V_P > V_Q$

Hence, work done will be negative.

23. Electrostatic potential energy,

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\sum_{i=1,2}\frac{q_1q_2}{r_{12}}$$

27. (a) The work done to bring the charge Q_3 from infinity to $\frac{r}{2}$,



$$W = U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r/2} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r/2} = \frac{1}{4\pi\varepsilon_0}\frac{2Q_3}{r}[Q_1 - Q_2]$$

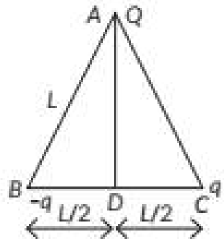
(b) Consider a point P at a distance x from Q_1 where work done is zero. Then

24. (i) $PE = \sum \frac{kq_iq_j}{r_{ij}}$

Potential energy of the arrangement is given by

$$U = \frac{kQ(-q)}{L} + \frac{kQq}{L} + \frac{k(-q)q}{L}$$

$$\therefore\ U = -\frac{kq^2}{L}$$

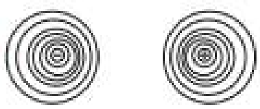


(ii) $AD = \sqrt{L^2 - \frac{L^2}{4}} = \frac{\sqrt{3}}{2}L$

Potential at D

$$V = \frac{k(-q)\times 2}{L} + \frac{kq\times 2}{L} + \frac{kQ\times 2}{\sqrt{3}L}$$

$$\therefore\ V = \frac{2kQ}{\sqrt{3}L}$$

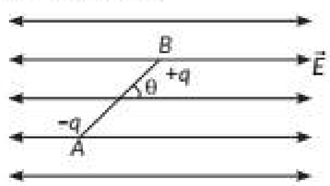
25. (a): 

(b) As $W = q_0\Delta V$

So, the Charge q_0 moves from $(l, 0, 0)$ to $(0, 0, 0)$ means along x-axis. x-axis in the equipotential line for a given system of charges, $\Delta V = 0$

So, $W = 0$

26. Consider an electric dipole having charges $+q$ and $-q$ is placed in external field $\vec{E}$.



Torque experienced by the dipole

$\tau = PE\sin\theta$

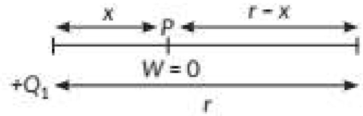
Potential energy $= \int_{\theta_1}^{\theta_2} \tau . d\theta = \int_{\theta_1}^{\theta_2} PE\sin\theta d\theta$

$$U = -PE[\cos\theta]_{\theta_1}^{\theta_2} = -PE(\cos\theta_2 - \cos\theta_1)$$

If $\theta_1 = 90°$, $\theta_2 = \theta$

$$U = -PE\ \cos\theta = -\vec{P}\cdot\vec{E}$$

Potential energy is minimum if θ is 90°.



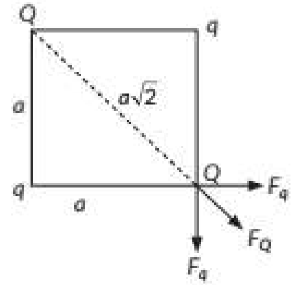
$\therefore$ Potential at P due to Q_1 = potential at P due to Q_2

$$\frac{kQ_1}{x} = \frac{kQ_2}{(r-x)} \Rightarrow (r-x)Q_1 = xQ_2$$

$$rQ_1 - xQ_1 = xQ_2 \Rightarrow rQ_1 = x(Q_1 + Q_2)$$

$$\Rightarrow\ x = \frac{rQ_1}{Q_1 + Q_2}$$

28. (a) Force on charge Q due to charge q.



$$F_q = \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{a^2}$$

Force on charge Q due to another charge Q,

$$F_Q = \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$$

Net force on charge Q is

$$F_{net} = F_Q + \sqrt{F_q^2 + F_q^2} = F_Q + F_q\sqrt{2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{2a^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{a^2}\sqrt{2}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{Q}{2} + \sqrt{2}\,q\right] \text{ along diagonal}$$

(b) Potential energy of the given system,

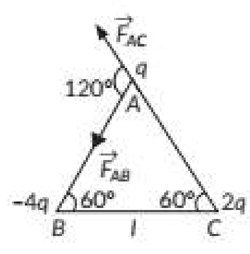
$$U = U_{qQ} + U_{Qq} + U_{qQ} + U_{Qq} + U_{qq} + U_{QQ} = 4U_{qQ} + U_{qq} + U_{QQ}$$

$$= \frac{4qQ}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}a)} + \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)}$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left[4qQ + \frac{q^2}{\sqrt{2}a} + \frac{Q^2}{\sqrt{2}a}\right]$$

29. (a) $F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q(4q)}{l^2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{l^2}$$



$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q(2q)}{l^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{l^2}$$

Angle between forces $\vec{F}_{AB}$ and $\vec{F}_{AC}$ is 120°.

Magnitude of resultant force,

$$F = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\cos 120°}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{l^2}\right)\sqrt{(4)^2 + (2)^2 + 2\times 4\times 2\times\left(\frac{-1}{2}\right)}$$

$$W = \int_{\theta_0}^{\theta} \tau_{ext}(\theta)d\theta = \int_{\theta_0}^{\theta} pE\sin\theta d\theta$$

$= pE[-\cos\theta]_{\theta_0}^{\theta}$

$= pE[-\cos\theta - (-\cos\theta_0)]$

$= pE[-\cos\theta + \cos\theta_0]$

$= pE[\cos\theta_0 - \cos\theta]$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\sqrt{16+4-8}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}(2\sqrt{3})$$

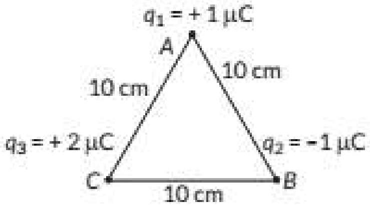
(b) Required work done = Change in potential energy of the system = $U_f - U_i$

$$= 0 - (U_{AB} + U_{BC} + U_{CA})$$

$$=\frac{-1}{4\pi\varepsilon_0 l}[q(-4q)+(-4q)(2q)+(q)(2q)]$$

$$=\frac{-1}{4\pi\varepsilon_0 l}[-4q^2-8q^2+2q^2]=\frac{10q^2}{4\pi\varepsilon_0 l}$$

30. The work done in bringing a charge q_1 from infinity to point A, $W_A = 0$.



Work done in bringing charge q_2 to point B from infinity

$$W_B=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}=\frac{1}{4\pi\varepsilon_0}\frac{-1\times10^{-12}}{10\times10^{-2}}$$

Work done in bringing a point charge q_3 to point C from infinity,

$$W_C=\frac{1}{4\pi\varepsilon_0}\frac{1\times2\times10^{-12}}{10\times10^{-2}}+\frac{1}{4\pi\varepsilon_0}\frac{2\times(-1)\times10^{-12}}{10\times10^{-2}}$$

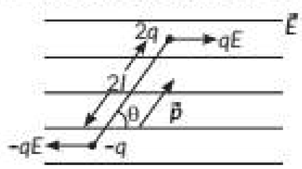
$\therefore$ Total work done, $W = W_A + W_B + W_C$

$$=\frac{1}{4\pi\epsilon_0}\left[0+\frac{-1\times10^{-12}}{10\times10^{-2}}+\frac{2\times10^{-12}}{10\times10^{-2}}+\frac{(-2)\times10^{-12}}{10\times10^{-2}}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{-1\times10^{-12}}{10\times10^{-2}}$$

$= 9 \times 10^9 \times (-0.1) \times 10^{-10} = -0.9 \times 10^{-1} = -0.09$ J

31. Consider a dipole with charge $-q$ and $+q$ seperated by a finite distance $2l$, placed in a uniform electric field $\vec{E}$. It experiences a torque $\vec{\tau}$ which tends to rotate it as shown in figure below,



$\vec{\tau}=\vec{p}\times\vec{E}=pE\sin\theta$

To neutralize this torque, let us assume an external torque $\vec{\tau}_{ext}$ be applied, which rotates it in the plane of the paper from angle q_0 to angle θ, without angular acceleration and at an infinitesimal angular speed.

Work done by the external torque.

This work done is stored as the potential energy of the system in the position when the dipole makes an angle θ with the electric field.

Assuming potential energy to be zero when $\theta = 90°$

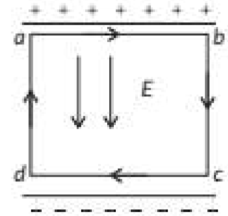
Putting $\theta_1 = 90°$, $U_1 = 0$

and $\theta_2 = \theta$, $U_2 = U$

$U - 0 = pE(\cos 90° - \cos\theta) \Rightarrow U = -pE\cos\theta$

In vector form, $U=-\vec{p}\cdot\vec{E}$

32. Electric field inside a parallel plate capacitor = E



Here, electric field is conservative. Work done by the conservative force in closed loop is zero.

So, required work done = 0.

33. The work done in bringing charge q_1 in the external electric field at a distance $\vec{r}_1 = q_1 V(r_1)$.

Work done in bringing charge q_2 in the external electric field at a distance $\vec{r}_2 = q_2 V(r_2)$.

The work done in moving q_2 against the force of q_1

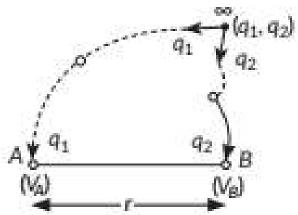
$$=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

$\therefore$ Potential energy of the system

$$q_1V(r_1)+q_2V(r_2)+\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$$

34. Potential energy of a system of two charges in an external field



$W_1 = q_1(V_A - 0)$

$= q_1V_A$

where V_A is the potential at point A due the external field.

Now work done in bringing charge q_2 to point B

$$W_2=q_2V_B+\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

where V_B is the potential due to the external field at point B.

Total work done in assembling the configuration of two charges in an electric field is

$W = W_1 + W_2$

$$\therefore W=q_1V_A+q_2V_B+\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

Key Points

- Work done only depends upon the initial and final position and is independent of path.

35. Since net force on electric dipole in uniform electric field is zero, so no work is done in moving the electric dipole in uniform electric field, however some work is done in rotating the dipole against the torque acting on it. So, small work done in rotating the dipole by an angle $d\theta$ in uniform electric field E is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

Hence, net work done in rotating the dipole from angle θ_i to θ_f in uniform electric field is

$$W = \int_{\theta_i}^{\theta_f} pE\sin\theta\, d\theta = pE\left[-\cos\theta\right]_{\theta_i}^{\theta_f}$$

or $\quad W = pE\,[-\cos\theta_f + \cos\theta_i] = pE\,[\cos\theta_i - \cos\theta_f]$

If initially, the dipole is placed at an angle $\theta_i = 90°$ to the direction of electric field, and is then rotated to the angle $\theta_f = \theta$, then net work done is

$$W = pE\,[\cos 90° - \cos\theta]$$

or $\quad W = -\,pE\cos\theta$

This gives the work done in rotating the dipole through an angle θ in uniform electric field, which gets stored in it in the form of potential energy *i.e.*,

$$U = -\,pE\cos\theta$$

This gives potential energy stored in electric dipole of moment p when placed in uniform electric field at an angle θ with its direction.

angle θ with its direction.

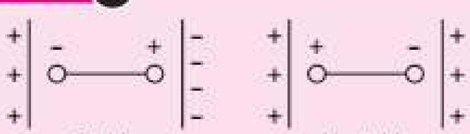
(i) When $\theta = 0°$, then $U_{min} = -pE$

So, potential energy of an electric dipole is minimum, when it is placed with its dipole moment p parallel to the direction of electric field E and so it is called its most stable equilibrium position.

(ii) When $\theta = 180°$, then $U_{max} = +\,pE$

So, potential energy of an electric dipole is maximum, when it is placed with its dipole moment p anti-parallel to the direction of electric field E and so it is called its most unstable equilibrium position.

Answer Tips

- 

Stable Unstable

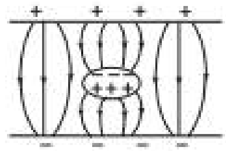
36. Let P be a point at distance r from the sheet.

$$W = q\cdot(V_P - V_\infty) \quad \text{...(i)}$$

$$-\frac{\sigma}{2\varepsilon_0}(r-\infty)=\infty \text{ or, } V_p - V_\infty = \infty$$

From eq. (i) $W = \infty$

37. **(d):** When an neutral conductor is placed between plates, a negative charge is induced on upper part and positive charge is induced on lower part of sphere so the correctly field lines are represented in (d).



38. The plate area of C_2 is greater than that of C_1. Since capacitance of a capacitor is directly proportional to the area of the plates,

$\therefore \quad C_2 > C_1$

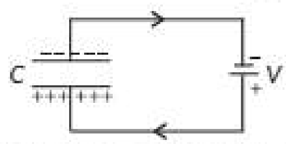
Now, $C = \dfrac{q}{V}$

Therefore, slope of a line ($=q/V$) is directly proportional to the capacitance of the capacitor, it represents. Since the slope of line A is more than that of B, line A represents C_2 and the line B represents C_1.

39. **(c):** The electric field between the oppositely charged plates of a capacitor in twice of that due to one plate. So, when the one plate is removed the electric force reduces to half of its earlier value. [1]

to half of its earlier value. [1]

40. Consider a parallel plate capacitor is connected across a battery as shown in figure.



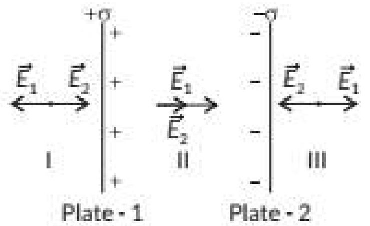
Then the electric current will flow through the circuit. As the electrons reach the plate, the insulating gap does not allow the electrons to move further; hence, positive charges develop on one side of the plate and negative charges develop on the other side of the plate. As the voltage begins to develop, the electric charges begins to resist the deposition of further charges. Thus the current flowing through the circuit gradually becomes less then zero till the voltage of the capacitor is exactly equal but opposite to the voltage of the battery. This is how capacitor gets charged.

41. Capacitor is based on the principle of electrostatic induction. The capacitance of an insulated conductor increases significantly by bringing an uncharged earthed conductor near to it. This combination forms parallel

Now, $V_P - V_\infty = -\int_\infty^r \vec{E}\cdot d\vec{r} = -\int_\infty^r E dr = -\int_\infty^r \left(\frac{\sigma}{2\varepsilon_0}\right) \cdot dr$

(Field from an infinitely large plane sheet of charge q is uniform and is given by $\frac{\sigma}{2\varepsilon_0}$).

$$-\frac{\sigma}{2\varepsilon_0}\int_\infty^r dr = -\frac{\sigma}{2\varepsilon_0}\cdot[r]_\infty^r$$



(i) In region I (outside)

$$E_I = E_2 - E_1 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(ii) In region II (inside)

$$E_{II} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

(iii) In region III (outside)

$$E_{III} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the region II *i.e.*, in the space between the plates, resultant electric field $\vec{E}_{II}$ is directed normal to plates, from positive to negative charge plate.

(b) The potential difference between the plates is

$$V = E_{II}.d = \frac{\sigma}{\varepsilon_0}d \quad \text{or} \quad V = \frac{Q}{A\varepsilon_0}d$$

(c) Capacitance of the capacitor so formed is

$$C = \frac{Q}{V} = \frac{Q}{Qd/A\varepsilon_0} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

42. (a): When they are connected in parallel, V is same

So, $\frac{q_1}{C_1} = \frac{q_2}{C_2}$

$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

43. (i) $\phi_1 = \frac{Q}{\varepsilon_0}, \phi_2 = \frac{3Q}{\varepsilon_0}$

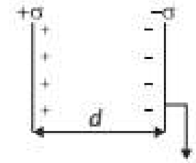
$$\frac{\phi_1}{\phi_2} = \frac{1}{3}$$

(ii) If a medium of dielectric constant 5 is filled in the space inside S_1, the flux inside S_1

$$\phi_1' = \frac{Q}{5\varepsilon_0} = \frac{\phi_1}{5}$$

44. (a): $C_1 = \frac{K\epsilon_0 A}{d}$

plate capacitor.



(a) Magnitude of electric field intensities

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

Now C'_1 and C'_2 are in series

$$\frac{1}{C_s} = \frac{d}{2\epsilon_0 K_2 A} + \frac{d}{2\epsilon_0 K_1 A}$$

$$C_s = \left(\frac{2K_1K_2}{K_1+K_2}\right)\frac{\epsilon_0 A}{d}$$

As $C_1 = C_s$

So, $K = \frac{2K_1K_2}{K_1+K_2}$

45. (i) Capacitance $C = \frac{\varepsilon_0 A}{d}$

$$= \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-4}} = 17.7\times10^{-11}\text{F}$$

(ii) Charge $Q = CV = 17.7 \times 10^{-11} \times 100$
$= 17.7 \times 10^{-9}$ C

(iii) $C' = KC$

$\therefore$ $Q' = KQ = 10.62 \times 10^{-8}$ C

Answer Tips

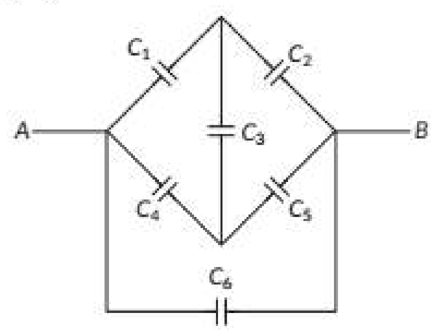
- When a dielectric slab of constant K is inserted between the plates of capacitors, the capacitance is increased by K times.

46. (i) As the bridge is balanced, so C_1 and C_2 are in series and C_4 and C_5 are in series.

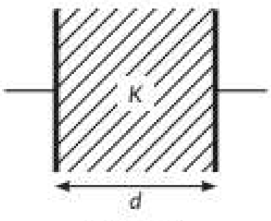
$$C_S = C_{S'} = \frac{C}{2}$$

and C_S and $C_{S'}$ are in parallel

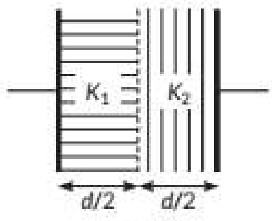
$$C_P = \frac{C}{2} + \frac{C}{2} = C$$



Now C_P and C_6 are in parallel



(Case 1)



(Case 2)

$$C_1' = \frac{K_1 \epsilon_0 A \times 2}{d}$$

$$C_2' = \frac{K_2 \epsilon_0 A \times 2}{d}$$

(a) Common potential, $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

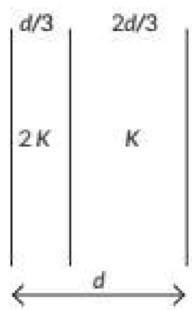
$$V = \frac{Q+0}{C+2C} = \frac{Q}{3C}$$

(b) Now charge on A, $Q_{A'} = CV = \frac{C \times Q}{3C} = \frac{Q}{3}$

Charge lost by $A = Q - Q_{A'} = Q - \frac{Q}{3} = \frac{2Q}{3}$

OR

(iii) $C_1 = \frac{2K\varepsilon_0 A \times 3}{d}$



$$C_2 = \frac{K\varepsilon_0 A \times 3}{2d}$$

Now both are in series

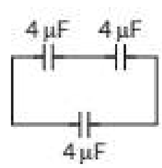
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{6K\varepsilon_0 A} + \frac{2d}{3K\varepsilon_0 A}$$

$$\frac{1}{C_S} = \frac{d+4d}{6K\varepsilon_0 A}$$

$$C_S = \frac{6K\varepsilon_0 A}{5d}$$

47. (c) : For the two series capacitors

$$C_{eq} = \frac{4 \times 4}{4+4}$$



$$C_{eq} = 2\,\mu\text{F}$$

Now, C_{eq} and 4 μF in parallel.

$C = 2 + 4 = 6\,\mu\text{F}$

48. (a) : Voltage, $V = 200$ V, $C_1 = 2\,\mu$F to $C_2 = X\,\mu$F

Decrease in energy, $\Delta U = 2 \times 10^{-2}$ J

$$C_{eq} = C + C = 2C$$

(ii) when dielectric is inserted,

As $E_0 = \frac{\sigma}{\varepsilon_0}$

$$E = \frac{\sigma}{K\varepsilon_0} = \frac{E_0}{K}$$

Thus, the electric field decreases.

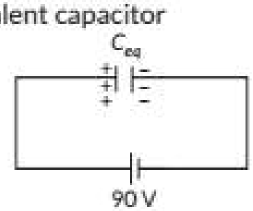
(iii) $A \rightarrow C, Q$

$V_A = Q/C$

$B \rightarrow 2C, Q = 0$

$V_B = 0$

Charge on equivalent capacitor



$$Q = C_{eq}V = \frac{60}{9} \times 10^{-6} \times 90$$

$Q = 600\,\mu$C

Charge on each capacitor is same as they are in series.

Now, potential drop across C_2

$$V_2 = \frac{Q}{C_2} = \frac{600 \times 10^{-6}}{30 \times 10^{-6}} = 20 \text{ volt}$$

Energy, $U = \frac{1}{2}C_2V_2^2$

$$U = \frac{1}{2} \times 30 \times 10^{-6} \times (20)^2 = 6 \times 10^{-3} \text{ joule}$$

50. Energy stored in a capacitor

$$= \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Capacitance of the (parallel) combination

$= C + C = 2C$

Here, total charge Q, remains the same.

$\therefore$ Initial energy (Single capacitor) $= \frac{1}{2}\frac{Q^2}{C}$

and final energy (Combined capacitor) $= \frac{1}{2}\frac{Q^2}{2C}$

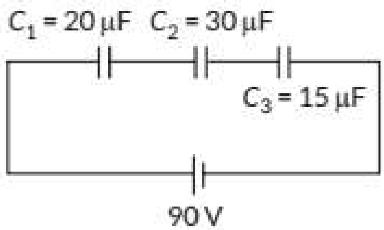
$$\therefore \quad \frac{\text{Final energy}}{\text{Initial energy}} = \frac{1}{2}$$

$$\Delta U=\frac{1}{2}C_1V^2-\frac{1}{2}C_2V^2$$

$$2\times10^{-2}=\frac{1}{2}\times200\times200(2-X)\times10^{-6}\ ;\ X=1\ \mu F$$

49. The equivalent capacitance (C_{eq}) of the circuit is given by

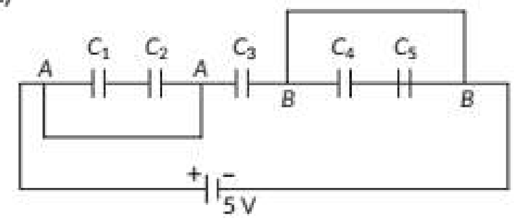
$$\frac{1}{C_{eq}}=\frac{1}{20}+\frac{1}{30}+\frac{1}{15}$$



$$\frac{1}{C_{eq}}=\frac{3+2+4}{60}$$

$$C_{eq}=\frac{60}{9}\mu F$$

51. (a)



A and B are short circuit therefore, effective capacitance is only C_3.

So the equivalent capacitance between A and B is $C_3 = 2\ \mu F$

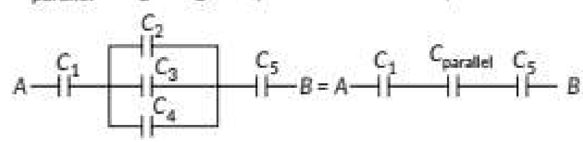
(b) Charge, $Q = CV = 2\ \mu F \times 5V = 10\ \mu C$

(c) Total energy stored

$$E=\frac{1}{2}CV^2=\frac{1}{2}\times2\ \mu F\times(5V)^2=25\ \mu J$$

52. (i) In the circuit C_2, C_3 and C_4 are in parallel

$\therefore$ $C_{parallel} = C_2 + C_3 + C_4 = 2 + 2 + 2 = 6\ \mu F$



$\therefore$ Equivalent capacitance between A and B is

$$\frac{1}{C_{equivalent}}=\frac{1}{C_1}+\frac{1}{C_{parallel}}+\frac{1}{C_5}$$

$$=\frac{1}{2}+\frac{1}{6}+\frac{1}{2}=\frac{3+1+3}{6}=\frac{7}{6}$$

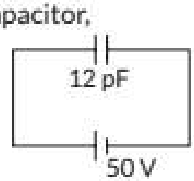
$\therefore$ $C_{equivalent} = \frac{6}{7} = 0.86\ \mu F$

(ii) $Q = C_{equivalent}V = 0.86 \times 7 = 6\ \mu C$.

$\therefore$ Energy, $E = \frac{1}{2}QV = \frac{1}{2} \times 6 \times 7 = 21$ J

53. Electrostatic energy stored in the capacitor,

$$U=\frac{1}{2}CV^2=\frac{1}{2}\times12\times10^{-12}\times(50)^2$$

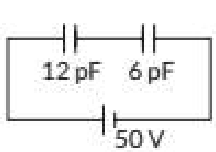


(As C = 12 pF, V = 50 V)

$U = 1.5 \times 10^{-8}$ J

When 6 pF is connected in series with 12 pF, charge stored across each capacitor,

$$Q=\frac{C_1C_2}{C_1+C_2}V$$



$$=\frac{12\times6\times10^{-24}}{(12+6)\times10^{-12}}\times50=200\text{ pC}$$

Now, potential difference across 12 pF is,

$$=\frac{Q}{C_1}=\frac{200\times10^{-12}}{12\times10^{-12}}=16.67\text{ V}$$

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, and the charge on an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to the battery.

Hence, potential V remains constant on it.

When the capacitors are filled with dielectric, their capacitance increases to KC. Therefore, energy stored in B changes to $Q^2/2KC$, where $Q = CV$ is the charge on B, which remains constant, and energy stored in A changes to $1/2\ KCV^2$, where V is the potential on A, which remains constant. Thus, the final total energy stored in the capacitors is

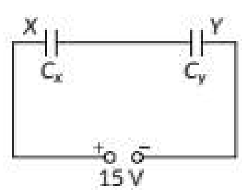
$$U_f=\frac{1}{2}\frac{(CV)^2}{KC}+\frac{1}{2}KCV^2=\frac{1}{2}CV^2\left(K+\frac{1}{K}\right)\qquad\text{...(ii)}$$

From Eqs. (i) and (ii), we find

$$\frac{U_i}{U_f}=\frac{2K}{K^2+1}$$

56. Here, $C_x=\frac{\varepsilon_0A}{d}$

$$C_y=\frac{\varepsilon_0\varepsilon_rA}{d}=\varepsilon_rC_x=4C_x$$



(i) C_x and C_y are in series, so equivalent capacitance is given by

Potential difference across 6 pF is,

$$=\frac{Q}{C_2}=\frac{200\times10^{-12}}{6\times10^{-12}}=33.33\,\text{V}$$

54. When two identical capacitors are in series,
Electrostatic energy,

$$U=\frac{1}{2}C_sV^2$$

As, $C_s=\frac{C_1C_2}{C_1+C_2}=\frac{12\times12}{12\times12}=6\,\text{pF};\ V=50\,\text{V}$

$$\therefore\ U_s=\frac{1}{2}\times6\times10^{-12}\times(50)^2=7.5\,\text{nJ}$$

When two identical capacitors are in parallel then,

Stored energy, $U_p=\frac{1}{2}C_pV^2$

As, $C_p = C_1 + C_2 = 12\text{ pF} + 12\text{ pF} = 24\times10^{-12}\text{ F}$

$$\therefore\ U_p=\frac{1}{2}\times24\times10^{-12}\times(50)^2=30\,\text{nJ}$$

Charge drawn from the battery when two identical capacitor are in series,
$Q_s = C_sV = 6\times10^{-12}\times50 = 300\text{ pC}$
Charge drawn from the battery when two capacitor are in parallel,
$Q_p = C_pV = 24\times10^{-12}\times50 = 1200\text{ pC}$

55. Initially, when the switch is closed, both the capacitors A and B are in parallel and, therefore, the energy stored in the capacitors is

$$U_i=2\times\frac{1}{2}CV^2=CV^2 \qquad \text{...(i)}$$

Since potential is same for parallel connection, the potential through 12 μF capacitor is also V. Hence, energy of 12 μF capacitor is

$$E_{12}=\frac{1}{2}\times12\times10^{-6}\times V^2=\frac{1}{2}\times12\times10^{-6}\times\frac{E}{3}\times10^6=2E$$

(ii) Since charge remains constant in series, the charge on 6 μF and 12 μF capacitors combined will be equal to the charge on 3 μF capacitor. Using the formula, $Q = CV$, we can write

$$(6+12)\times10^{-6}\times V = 3\times10^{-6}\times V'$$

$V' = 6\text{ V}$
Using (i) and squaring both sides, we get
$V'^2 = 12E\times10^6$

$$\therefore\ E_3=\frac{1}{2}\times3\times10^{-6}\times12E\times10^6=18E$$

(iii) Total energy drawn from battery is

$$E_{total} = E + E_{12} + E_3 = E + 2E + 18E = 21E$$

58. When two capacitors C_1 and C_2 are in parallel,
Equivalent capacitance, $C_p = C_1 + C_2$

Energy stored, $U_p = \frac{1}{2}C_pV^2 = \frac{1}{2}(C_1+C_2)V^2$

$$C=\frac{C_x\times C_y}{C_x+C_y}$$

$$\Rightarrow\ 4=\frac{C_x\times4C_x}{C_x+4C_x} \qquad (\because C = 4\,\mu\text{F})$$

$$\Rightarrow\ 4=\frac{4C_x}{5}\ \therefore C_x = 5\,\mu\text{F}$$

and $C_y = 4C_x = 20\,\mu\text{F}$

(ii) Charge on each capacitor, $Q = CV$
$Q = 4\times10^{-6}\times15 = 60\times10^{-6}\text{ C}$

Potential difference between the plates of X,

$$V_x=\frac{Q}{C_x}=\frac{60\times10^{-6}}{5\times10^{-6}}=12\,\text{V}$$

Potential difference between the plates of Y,
$V_y = V - V_x = 15 - 12 = 3\text{ V}$

(iii) Ratio of electrostatic energy stored,

$$\frac{U_x}{U_y}=\frac{\frac{Q^2}{2C_x}}{\frac{Q^2}{2C_y}}=\frac{C_y}{C_x}=\frac{4C_x}{C_x}=4$$

57. (i) Given that energy of the 6 μF capacitor is E
Let V be the potential difference along the capacitor of capacitance 6 μF.

Since, $\frac{1}{2}CV^2=E\ \ \therefore\ \frac{1}{2}\times6\times10^{-6}\times V^2=E$

$$\Rightarrow V^2=\frac{E}{3}\times10^6 \qquad \text{...(i)}$$

When dielectric is inserted, $E=\frac{E_0}{K}=\frac{\sigma}{E_0K}$

Potential difference, $V=Ed=\frac{\sigma d}{E_0K}$

Now, $C=\frac{Q}{V}=\frac{QE_0K}{\sigma d}=\frac{QE_0KA}{Qd}$

$$C=\frac{KE_0A}{d}$$

(ii) As the surface of sphere is equipotential, so the work done in moving the charge from one point to the other is zero.

$$W=q\cdot\Delta V=0$$

(iii) $\vec{P}=\chi\vec{E}$

60. (i) Consider an electric dipole having changes $+q$ and $-q$ is placed in external field $\vec{E}$.
Torque experienced by the dipole
$\tau = PE\sin\theta$

Here, $U_p = 0.25$ J, $V = 100$ V

$$C_1+C_2=\frac{2U_p}{V^2}=\frac{2\times0.25}{(100)^2}$$

$\therefore \quad C_1 + C_2 = 5 \times 10^{-5}$ ...(i)

When C_1 and C_2 are connected in series

Equivalent capacitance, $C_s = \dfrac{C_1C_2}{C_1+C_2}$

Energy stored, $U_s = \dfrac{1}{2}C_sV^2 = \dfrac{1}{2}\left(\dfrac{C_1C_2}{C_1+C_2}\right)V^2$

Here, $U_s = 0.045$ J

$$\therefore \quad C_1C_2=\frac{2U_s(C_1+C_2)}{V^2}$$

$$=\frac{2\times0.045\times5\times10^{-5}}{10^4}=4.5\times10^{-10}$$

$$C_1-C_2=\sqrt{(C_1+C_2)^2-4C_1C_2}=\sqrt{(5\times10^{-5})^2-4\times4.5\times10^{-10}}$$

$C_1 - C_2 = 2.64 \times 10^{-5}$ ...(ii)

Solving eq. (i) and (ii), we get

$C_1 = 38.2\ \mu$F, $C_2 = 11.8\ \mu$F

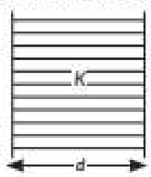
When capacitors are connected in parallel they have different amount of charge and given by

$Q_1 = C_1V = 38.2 \times 10^{-6} \times 100 = 38.2 \times 10^{-4}$ C

$Q_2 = C_2V = 11.8 \times 10^{-6} \times 100 = 11.8 \times 10^{-4}$ C.

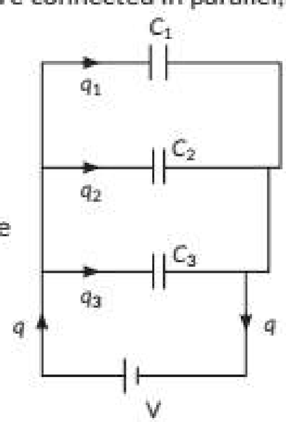
59. (i) (A) dielectric material gets polarized when it is placed is an external electric field. The field produced due to the polarization of material reduce the effect of external electric field. Hence, the electric field inside a dielectric decreases.

(B) Let the electric is filled between the plates of a capacitor.



The surface charge density is σ. The electric field between the plate when no dielectric is inserted, $E_0 = \dfrac{\sigma}{E_0}$

61. (a) (i) When the capacitors are connected in parallel, so



$q_1 = C_1 V, \quad q_2 = C_2 V, \quad q_3 = C_3 V$

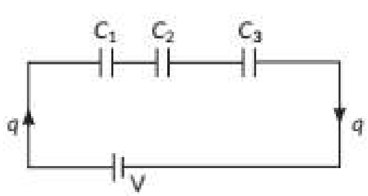
$q = q_1 + q_2 + q_3$

$C_P V = C_1 V + C_2 V + C_3 V$

$C_p = C_1 + C_1 + C_3$

(ii) When the capacitors are connected in series, so

$$V_1=\frac{q}{C_1}, V_2=\frac{q}{C_2}, V_3=\frac{q}{C_3}$$



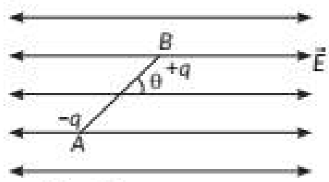
Potential energy $= \int_{\theta_1}^{\theta_2}\tau.d\theta=\int_{\theta_1}^{\theta_2}PE\sin\theta d\theta$

$$U=-PE[\cos\theta]_{\theta_1}^{\theta_2}=-PE(\cos\theta_2-\cos\theta_1)$$

If $\theta_1 = 90°$, $\theta_2 = \theta$

$$U=-PE\cos\theta=-\vec{P}\cdot\vec{E}$$

Potential energy is minimum if θ is 90°.



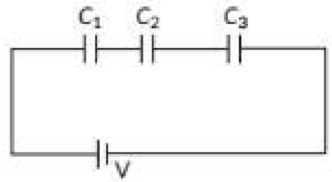
(ii) In series combination,

$$\frac{1}{C_{eff}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_2}$$

Now

$$U=\frac{1}{2}\frac{Q^2}{C_1}+\frac{1}{2}\frac{Q^2}{C_2}+\frac{1}{2}\frac{Q^2}{C_3}$$

$U = U_1 + U_2 + U_3$



(iii) When battery is disconnected then charge q remains same.

Capacitance, $C' = \dfrac{\epsilon_0 A}{2d} = \dfrac{C}{2}$

(ii) Electric field, $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{q}{\epsilon_0 A}$

It remains unchanged

(ii) Since energy stored in the capacitor

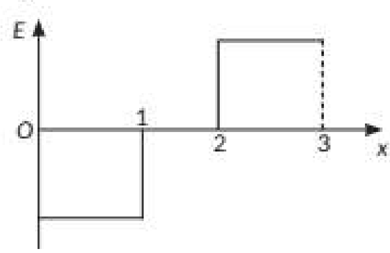
$$U=\frac{Q^2}{2C}=\frac{Q^2d}{2\epsilon_0 A}$$

Similarly $U' = \dfrac{Q}{2C'} = \dfrac{Q^2d}{2K\epsilon_0 A} = \dfrac{2U}{K}$

as $1 < K < 2$

So, the energy stored between the plates increases.

(iii) As $E = \dfrac{-d_v}{d_x}$, so the graph is



$V = V_1 + V_2 + V_3$

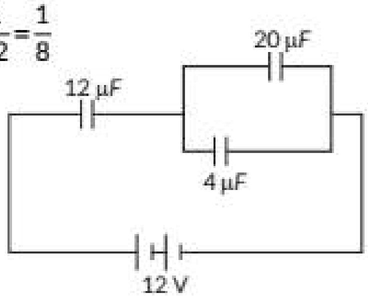
$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

(b) 20 μF and 4 μF is in parallel.

$C_p = 20 + 4 = 24\,\mu F$

Now $\frac{1}{C_s} = \frac{1}{C_p} + \frac{1}{12} = \frac{1}{24} + \frac{1}{12} = \frac{1}{8}$



$C_s = 8\,\mu F$

$q = C_s V = 8 \times 12 = 96\,\mu C$

$V_{12\mu}F = \frac{q}{C_1} = \frac{16}{12} = 8V$

So, $V_{4\mu}F = 12 - 8 = 4V$

Now, energy stored in 12 μF

$V = \frac{1}{2} \times C_{12} \times V_{12}^2 = \frac{1}{2} \times 12 \times 8 \times 8 = 384\,\mu J$

62. (a) Charging of capacitor with dc battery whenever parallel plate capacitor in charge in dc source, plates start acquiring charge in accordance with the terminals of the battery till potential difference across the plate becomes equal to terminal potential of dc battery.

(b) (i) The electric field between the plates of parallel plate capacitor

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{\theta}{\epsilon_0 A}$$

if dielectric is inserted

$$E' = \frac{\theta}{\epsilon_0 AK} = \frac{E_0}{K}$$

So, the electric field intensity decrease to $\frac{1}{K}$ times.

63. Energy stored in a capacitor

$= \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Capacitance of the (parallel) combination

$= C + C = 2C$

Here, total charge Q, remains the same.

$\therefore$ Initial energy (Single capacitor) $= \frac{1}{2}\frac{Q^2}{C}$

and final energy (Combined capacitor) $= \frac{1}{2}\frac{Q^2}{2C}$

$\therefore \frac{\text{Final energy}}{\text{Initial energy}} = \frac{1}{2}$

64. (i) The electric field between the plates is

$$E = \frac{V}{d}$$

The distance between plates is doubled, $d = 2d$

$\therefore E' = \frac{V'}{d'} = \left(\frac{V}{K}\right) \times \frac{1}{2d} = \frac{1}{2}\left(\frac{E}{K}\right)$

Therefore, if the distance between the plates is double, the electric field will reduce to one half.

(ii) As the capacitance of the capacitor

$$C' = \frac{\varepsilon_0 KA}{d'} = \frac{\varepsilon_0 KA}{2d} = \frac{1}{2}C$$

Energy stored in the capacitor is $U = \frac{Q^2}{2C}$

New energy, $U' = \frac{Q^2}{2C'} = \frac{Q^2}{2(1/2)C} = 2\left(\frac{Q^2}{2C}\right) = 2U$

Therefore, when the distance between the plates is doubled, the capacitance reduces to half and the energy stored in the capacitor becomes double.

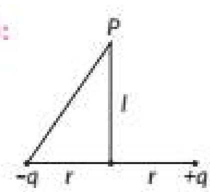
65. Given $\frac{C_1}{C_2} = \frac{1}{2}$ or $C_2 = 2C_1$

In parallel, $C_P = C_1 + C_2 = C_1 + 2C_1 = 3C_1$

In series, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{2C_1} = \frac{2+1}{2C_1} = \frac{3}{2C_1}$

or $C_S = \frac{2}{3}C_1$

3. (d):

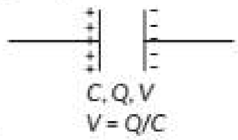


Given $U_S = U_P$

$$\frac{1}{2}C_S V_S^2 = \frac{1}{2}C_P V_P^2 \text{ or } \frac{2}{3}C_1 V_S^2 = 3C_1 V_P^2$$

or $\frac{V_S^2}{V_P^2} = \frac{9}{2}$ or $\frac{V_S}{V_P} = \frac{3}{\sqrt{2}}$

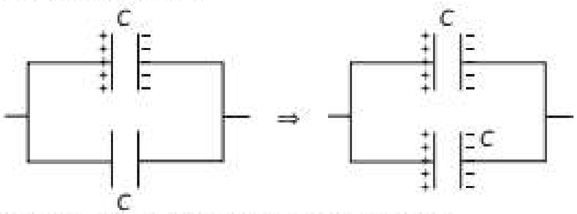
66. Let fully charge capacitor C has charge Q.



Energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Now, the charged capacitor is connected to identical uncharged capacitor.



The two capacitor will have same potential.

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q+0}{2C} = \frac{Q}{2C}$$

Now, total energy

$$U' = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$$

$$U' = \frac{1}{2}C\left(\frac{Q}{2C}\right)^2 + \frac{1}{2}C\left(\frac{Q}{2C}\right)^2 = \frac{Q^2}{4C}$$

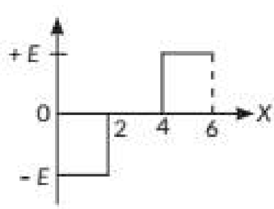
So, $U > U'$

Energy lost as heat during charging the another capacitor.

$$U - U' = \frac{Q^2}{2C} - \frac{Q^2}{4C} = \frac{Q^2}{4C}$$

CBSE Sample Questions

1. (a): As, $E = -\frac{dV}{dx}$. Hence, the graph of electric field E as a function of 'x' will be shown as:

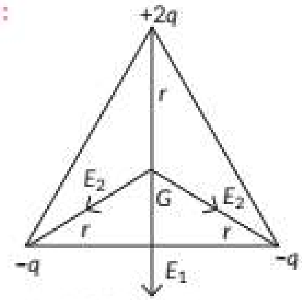


(1)

2. (d): Both the electric potential and electric field achieve a maximum magnitude at B. (0.77)

Potential at 'P' is, $V_P = \frac{K(-q)}{\sqrt{r^2+l^2}} + \frac{K(q)}{\sqrt{r^2+l^2}} = 0$ (1)

4. (c):



Net electric field intensity at centre G, $E \neq 0$

Net potential at G,

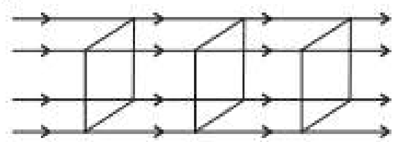
$$V = \frac{k \times 2q}{r} - \frac{kq}{r} - \frac{kq}{r} \quad (0.77)$$

$\therefore \quad V = 0$

5. (c): In uniform electric field, equipotential surfaces are never concentric spheres as they can never intersect but perpendicular to electric field lines. (0.77)

6. (b): Electric field is always at right angle to equipotential surface because there is no potential gradient along any direction parallel to the surface and so an electric field parallel to surface. (0.77)

7. Equipotential surfaces in a constant electric field in Z-direction.



(1)

For constant electric field

Electric field as gradient of potential consider a point charge $+q$ placed at point $\bar{O}$. Suppose that V and $V+\delta V$ are electrostatic potential at points A and B, where distance from the charge $+q$ are r and $r - \delta r$ respectively.

$$(V + \delta V) = V + \frac{\delta W}{q_0}$$

$$\delta V = \frac{\delta W}{q_0} \quad \text{...(i)}$$

If $\vec{E}$ is electric field at point P due to charge $+q$ placed at point $\bar{O}$, then the test charge q_0 experiences a force equal to $q_0\vec{E}$ and the external force required to move the test charge against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0\vec{E}$$

Therefore, work done to move the test charge through an infinitesimally small displacement $\overline{PQ} = \overline{\delta l}$ is given by

$$\Delta W = \vec{F} \cdot \overline{\delta l} = (-q\vec{E}) \cdot \overline{\delta l} = -q_0 E \delta l \cos 180° = q_0 E \delta l$$

As the distance r decreases in the direction of δl, then

$\delta W = -q_0 E\, \delta r$

$$\frac{\delta W}{q_0} = -E\, \delta r \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$$\delta V = -E\, \delta r;\ E = -\frac{\delta V}{\delta r} \quad (1)$$

8. (b): $W = pE(\cos\theta_1 - \cos\theta_2)$

As, $\theta_1 = 0°$ and $\theta_2 = 90°$

$\therefore \quad W = pE(\cos 0° - \cos 90°)$

$= pE(1 - 0) = pE$ (0.77)

9. (c): Electrons move from a region of low potential to high potential. (0.77)

10. The work done in bringing charge q_1 in the external electric field at a distance $\vec{r}_1 = q_1 V(r_1)$

work done in bringing charge q_2 in the external electric field at a distance $\vec{r}_2 = q_2 V(r_2)$

The work done in moving q_2 against the force of q_1

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

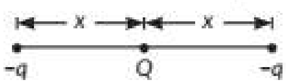
where r_{12} is the distance between q_1 and q_2.

$\therefore$ Potential energy of the system

$$q_1 V(r_1) + q_2 V(r_2) + \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

Therefore, electric field at a point is equal to the negative gradient of the electrostatic potential at that point. (2)

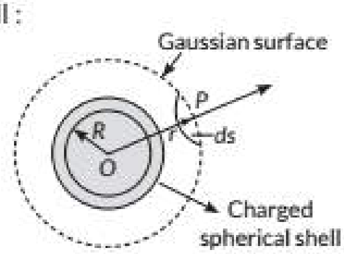
11. (a)



$$\frac{k(-q)Q}{x} + \frac{kQ(-q)}{x} + \frac{k(-q)(-q)}{2x} = 0$$

$$\frac{-2kqQ}{x} + \frac{kq^2}{2x} = 0 \text{ or } \frac{kq^2}{2x} = \frac{2kqQ}{x};\ q = 4Q \text{ or } \frac{Q}{q} = \frac{1}{4} \quad (2)$$

(b) Electric field due to a uniformly charged thin spherical shell :



(i) When point P lies outside the spherical shell : Suppose

$\therefore$ Total electric flux through the Gaussian surface is given by

$$\phi = \oint E ds = E \oint ds$$

Now, $\oint ds = 4\pi r^2$

$\therefore \quad \phi = E \times 4\pi r^2$ ...(i)

Since the charge enclosed by the Gaussian surface is q, according to the Gauss's theorem,

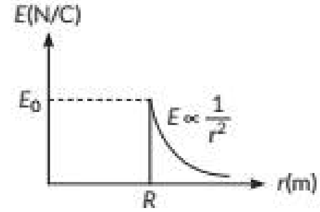
$$\phi = \frac{q}{\epsilon_0} \quad \text{...(ii)}$$

From equation (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \text{ (for } r > R)$$

(ii) A graph showing the variation of electric field as a function of r is shown below:



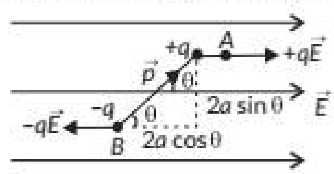
(3)

12. (a) A pair of equal and opposite charges separated by a small vector distance is called an electric dipole. An ideal dipole consists of two very very large charges $+q$ and $-q$ separated by a very very small distance. An ideal dipole has almost no size.

Water molecule is an example of electric dipole. (1)

(b) Torque on a dipole in uniform electric field:

When electric dipole is placed in a uniform electric field, its two charges experience equal and opposite forces, which cancel each other and hence net force on an electric dipole in a uniform electric field is zero.



However these forces are not collinear, so they give rise to some torque on the dipole given by

Torque = Magnitude of either force

× Perpendicular distance between them

$$\tau = Fr_1 = qE.2a \sin\theta = q2a. E \sin\theta$$

that we have to calculate field at point P at a distance r ($r > R$) from its centre. Draw Gaussian surface through point P so as to enclose the charged spherical shell. Gaussian surface is a spherical surface of radius r and centre O.

Let $\vec{E}$ be the electric field at point P, then the electric flux through area element of area $\vec{ds}$ is given by

$$d\phi = \vec{E} \cdot \vec{ds}$$

Since $\vec{ds}$ is also along normal to the surface

$$d\phi = E\, ds$$

or $\tau = pE \sin\theta$

where θ is the angle between the directions of $\vec{p}$ and $\vec{E}$.

In vectorial form, $\vec{\tau} = \vec{p} \times \vec{E}$

(i) When $\theta = 0°$ or $180°$ then $\tau_{min} = 0$

(ii) When $\theta = 90°$ then $\tau_{max} = pE$ **(1)**

Thus, torque on a dipole tends to align it in the direction of uniform electric field.

If the field is not uniform in that condition the net force on electric dipole is not zero. **(1)**

When $\theta = 0$; $\tau = 0$ and $\vec{p}$ and $\vec{E}$ are parallel and the dipole is in a position of stable equilibrium.

(c) Torque $\tau = PE\sin\theta = Ql\sin\theta$ ...(i)

Here l is the length of the dipole, Q is the charge and E is the electric field.

Therefore Q = Torque/$E\sin(\theta)$

$$= 8\sqrt{3}/(2 \times 10^{-2})(10^5)\frac{\sqrt{3}}{2} = 8 \times 10^{-3}\text{ C} \quad \textbf{(1)}$$

Potential energy, $U = -PE\cos\theta = -Ql\cos\theta$ ...(ii)

Divide equation (i) by (ii),

$$\frac{\tau}{U} = \frac{Ql\sin\theta}{-Ql\cos\theta} \text{ (where } P = Ql)$$

$$\frac{\tau}{U} = -\tan\theta \Rightarrow U = \frac{\tau}{-\tan 60°} = \frac{-8\sqrt{3}}{\sqrt{3}} = -8\text{ J} \quad \textbf{(1)}$$

13. (b): As the three capacitors are joined in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6}$$

$\therefore$ $C = 1\,\mu$F **(0.77)**

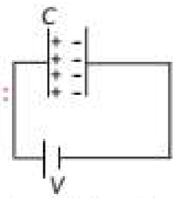
14. (b): $\therefore F_p = F_e$, $\therefore$ $F = qE$

$E = q$ = same

Now, P.E. = $qV(r)$

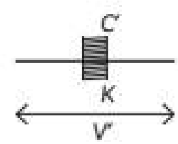
$\therefore$ $(\text{P.E.})_p > (\text{P.E.})_e$ **(0.77)**

15. (c):



$\therefore$ When battery is disconnected

Q = Charge remains constant



$C' = KC$

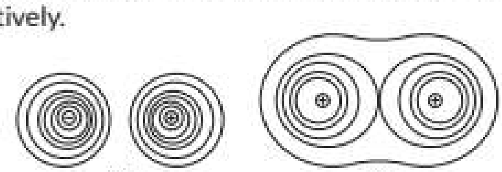
$Q' = C'V'$

$Q = C'V'$

$Q = KCV'$

$\therefore$ $V' = \dfrac{Q}{KC} = \dfrac{V}{K}$ $\left(\because V = \dfrac{Q}{C}\right)$ **(0.77)**

16. (a) Equipotential surfaces for a dipole and two identical positive charges, are shown in figure (i) and (ii) respectively.



(i) (ii) **(2)**

(b) Here, $A = 6 \times 10^{-3}\text{ m}^2$, $d = 3\text{ mm} = 3 \times 10^{-3}\text{ m}$

(i) Capacitance,

$$C = \frac{\epsilon_0 A}{d} = \left(\frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}\right) = 17.7 \times 10^{-12}\text{ F}$$

(ii) Charge, $Q = CV = 17.7 \times 10^{-12} \times 100$

$= 17.7 \times 10^{-10}$ C

(iii) New charge, $Q' = kQ = 6 \times 17.7 \times 10^{-10}$

$= 1.062 \times 10^{-8}$ C **(3)**